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On Radio Scattering by Dielectric Turbulence

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ON RADIO SCATTERING BY DIELECTRIC TURBULENCE

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September, 1956

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Abstract

A unified formulation of radio scattering theory in the language of random functions is given. The problem of probabilizing atmospheric dielectric noise is discussed in detail. The amplitude of the received scattered signal is a complex gaussian random function of time, with mean zero. Its statistical structure is therefore completely specified by its covariance, which is simply related to a mixed spectral density of the scattering dielectric noise. Local properties of second-order random functions are defined and discussed, and certain gaps in the literature are pointed out.

1. Introduction

In recent year it has been discovered that the scattering of radio waves by weak fluctuations of dielectric constant in the earth's atmosphere (both troposphere and ionosphere) can serve as a practical means of radio communication. The phrase dielectric noise is a convenient and suggestive designation for these fluctuations. There is little doubt that most dielectric noise in the atmosphere is produced by the action of turbulence at large Reynolds number. When we wish to emphasize its turbulent origin, we shall refer to dielectric noise as dielectric turbulence, borrowing Eckart's phrase[1].

The present paper is concerned with framing the radio scattering problem in its natural language, namely the theory of random functions. With few exceptions, this matter has received only a desultory treatment in the literature. The detailed structure of atmospheric dielectric noise is still very much in issue. Although for the purposes of this paper we do not have to be too explicit about the microstructure of dielectric noise, we assume that it is produced by turbulent mixing of dielectric constant inhomogeneities, and is described by the Kolmogoroff-Obukhoff structure law. The reasons why we favor this model of dielectric turbulence have been given elsewhere[2],*.

2. Preliminaries on random functions

The natural tool for probabilizing the radio scattering problem is the theory of random functions, treated in the books of Doob[6], Loève[7], and Blanc-Lapierre and Fortet[8], to which the reader is referred for many details and occasional undefined terms. We review here only those aspects of the theory which are indispensable for our present purposes.

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* The locally isotropic pressure field intrinsic to turbulent flow is a negligible source of atmospheric dielectric noise, as shown by Villars and Weisskopf[3], and Gallet[4]. In this connection, it should be remarked that the suggestion made by the author[5], that dependence between Fourier coefficients of the velocity field may greatly enhance turbulent pressure fluctuations, is fallacious.

In what follows, $\vec{r}$ denotes the position vector, referred to an arbitrary origin fixed with respect to the earth, and t is the time. If for every value of $\vec{r}$ and t, the quantity $f(\vec{r},t)$, in general complex,[*] is a random variable, $f(\vec{r},t)$ is said to be a random function or (random) process, of space and time. Let $f(\vec{r},t,\alpha)$ be an arbitrary realization or sample function of the process $f(\vec{r},t)$, where α is a point in a sample space S which indexes the realizations of $f(\vec{r},t)$. The statistical structure of $f(\vec{r},t)$ is specified by a probability measure μ, defined on the sets of S, or, equivalently, by the joint distribution of any n random variables $f(\vec{r}_1,t_1),\ldots,f(\vec{r}_n,t_n)$. Ensemble averages, so called to distinguish them from space and time averages, will be denoted by angular parentheses. They are integrals over S with respect to the measure μ, or, equivalently, integrals with respect to appropriate distribution functions, derived from the measure μ in a familiar manner (see [6], p. 10). For example

$$< f(\vec{r},t) > = \int_S f(\vec{r},t,\alpha)d\mu(\alpha),$$

$$< f(\vec{r},t)f^*(\vec{r}\,',t') > = \int_S f(\vec{r},t,\alpha)f^*(\vec{r}\,',t',\alpha)d\mu(\alpha),$$

where the asterisk denotes the complex conjugate.

The random function $f(\vec{r},t)$ is said to be second-order if

$$(1) \qquad < |f(\vec{r},t)|^2 > \; < \infty ,$$

for every $\vec{r}$ and t. Equation (1) implies that the mean $< f(\vec{r},t)>$ and the covariance $< f(\vec{r},t)f^*(\vec{r}\,',t') >$ exist for every $\vec{r}$, $\vec{r}\,'$, t, and t'. It is to be noted that in radio scattering, second-order properties of the received signal, e.g., its average power, its envelope correlation, etc., are of primary importance, as are the second-order properties of the dielectric noise, specified by its covariance.

---

[*] By a complex random variable (function) is meant a pair of real random variables (functions), namely its real and imaginary parts (see [7], p. 152).

The relevance of the theory of second-order random functions in physical problems like radio scattering stems from a natural preoccupation with power-like quantities.

If the distribution of the n random variables

$$f(\vec{r}_1 + \vec{h},\ t_1 + k),\ \ldots,\ f(\vec{r}_n + \vec{h},\ t_n + k) \tag{2}$$

is independent of $\vec{h}$ and k, for all $\vec{r}_1, t_1, \ldots, \vec{r}_n, t_n, \vec{h}, k$, and n, then $f(\vec{r},t)$ is said to be *stationary* in $\vec{r}$ and t. Stationarity in $\vec{r}$ and t separately is defined in the obvious way, and will sometimes be called r-stationarity and t-stationarity. If the stipulation made in connection with (2) is true for the values $n = 1$ and $n = 2$, without reference to whether it is true for all n, $f(\vec{r},t)$ is said to be *second-order stationary* or to have a *stationary coveriance*. Second-order stationarity is the case of most interest to us here, in keeping with our preoccupation with the second-order properties of $f(\vec{r},t)$.

Stationarity as just defined does not allow for the trends exhibited by most of the random processes that occur in nature. A natural way to take such trends into account is to introduce the weaker concept of *local stationarity*, which may be qualitatively defined as follows*. Suppose that for all n, $\vec{r}_1, t_1, \ldots, \vec{r}_n, t_n$, the distribution of the n random variables (2) is independent of $\vec{h}$ and k to a good approximation, provided that the points $(\vec{r}_1,t_1),\ldots,(\vec{r}_n,t_n)$ and their translates $(\vec{r}_1+\vec{h},t_1+k),\ldots,(\vec{r}_n+\vec{h},t_n+k)$ all lie in a certain domain D. Then D is said to be a *domain of local stationarity* of $f(\vec{r},t)$, and $f(\vec{r},t)$ is said to be locally stationary in D. Suppose now that the infinite space-time domain can be divided into non-overlapping domains of local stationarity of $f(\vec{r},t)$, and let $D(\vec{r},t)$ be the domain of local stationarity containing the point $D(\vec{r},t)$. This state of affairs will be summarized by saying that $f(\vec{r},t)$ is locally stationary in the (space-time) domains $D(\vec{r},t)$. The definitions of local stationarity in $\vec{r}$ and t are apparent, and involve, correspondingly, space and time domains of local stationarity. In particular, if $f(\vec{r},t)$ is locally stationary in the domains $D(\vec{r},t)$, then for every fixed t, $f(\vec{r},t)$ is locally r-stationary in the space domains $D_t(\vec{r})$, defined as the projections on the x,y,z axes of the domains $D(\vec{r},t)$; similarly, for every fixed $\vec{r}$, $f(\vec{r},t)$ is locally t-stationary in the time domains $D_r(t)$, defined as the projections on the t axis of the domains $D(\vec{r},t)$.

---

* The definition is qualitative in the sense that it contains the loose phrase " to a good approximation" , and the correspondingly loose concept of " domain of local stationarity" , but is adequate for the purposes of this paper. It is patterned after the definition of local homogeneity given by Kolmogoroff[9], in his first paper on the statistical theory of turbulence.

Suppose $f(\vec{r},t)$ is locally stationary in the domains $D(\vec{r},t)$. If for all $\vec{r}$ and t, $D(\vec{r},t)$ is the infinite space-time domain, then the definition of local stationarity reduces to the ordinary definition of stationarity, which we may call *global* stationarity, to distinguish it from *local* stationarity. For $n = 1$ and $n = 2$, we have a definition of second-order local stationarity or of locally stationary covariance, analogous to the corresponding definition for global stationarity. Again, it is the case of second-order local stationarity which is of most interest to us here. Following the usual heuristics, we shall deal with a random function $f(\vec{r},t)$ which has a locally stationary covariance in the domains $D(\vec{r},t)$ by approximating it by "pieces" of suitable random functions with globally stationary covariances. The piece used to approximate $f(\vec{r},t)$ in the domain $f(\vec{r},t)$ is, of course, taken from a globally stationary random function which has substantially the same covariance in $D(\vec{r},t)$ as $f(\vec{r},t)$. The qualitative nature of this approximation scheme parallels the inherently qualitative nature of our definition of local stationarity, but it is adequate for the purposes of this paper. A more rigorous treatment of random functions with locally stationary covariances will be presented elsewhere.

From a given random function $f(\vec{r},t)$, other random functions can be derived in a variety of ways, two of which we illustrate here. One consists in assigning a function of $\vec{r}$ and t to every sample function $f(\vec{r},t,\alpha)$, by a transformation

$$(3) \qquad T\, f(\vec{r},t,\alpha) = g(\vec{r},t,\alpha) \;;$$

we then define a random function of space and time $g(\vec{r},t)$, which has the sample functions $g(\vec{r},t,\alpha)$, and the same sample space S and probability measure μ as $f(\vec{r},t)$. The other way consists in assigning a function of t only to every sample function $f(\vec{r},t,\alpha)$ by a transformation

$$(4) \qquad T\, f(\vec{r},t,\alpha) = g(t,\alpha);$$

we then define a random function of time $g(t)$, which has the sample functions $g(t,\alpha)$ and the same sample space S and probability measure μ as $f(\vec{r},t)$.

As an example of the first transformation, we center the sample functions of $f(\vec{r},t)$ about their mean. As an example of the second transformation, we multiply each sample function by $\exp(i\vec{K}\cdot\vec{r})$ and integrate the product with respect to $\vec{r}$ over a finite region of space. As we shall see below, this example is a special case of the transformation which relates the dielectric noise responsible for radio scattering to the received scattered signal. Of special importance is the common case where the transformation T is homogeneous with respect to one or both of the variables $\vec{r}$ and t, either everywhere or in certain domains. For example, if equation (3) implies that

$$T\, f(\vec{r}-\vec{r}_o,t-t_o,\alpha) = g(\vec{r}-\vec{r}_o,t-t_o,\alpha)$$

for all α, $\vec{r}_o$, and t_o, then T is said to be (globally) homogeneous with respect to $\vec{r}$ and t (see [8], p. 19). If equation (4) implies that

$$T\, f(\vec{r}, t-t_o, \alpha) = g(t-t_o, \alpha)$$

for all α, provided t_o lies in a certain domain D(t) containing the point t, then T is said to be locally homogeneous with respect to t in the domain D(t). An immediate consequence of the homogeneity property is that one can infer the stationarily (global or local) of homogeneous transforms of a random function from the stationarity of the random function itself, as illustrated in the next section. (For a statement of the homogeneity property in the language of time-invariant filters, see Laning and Battin, [10], p. 120.)

In technological applications of the theory of (globally) stationary random functions, it is necessary at some point to invoke an ergodic property*, i.e., to identify ensemble averages with suitable experimentally available averages (e.g., space averages, time averages, etc.). Analogously, a local ergodic property is needed when dealing with locally stationary random functions, but one must be more careful about assuming its validity (see Sect. 2). A definition of such a property which is suitable for our purposes is the following. Let the random function f(x) be second-order locally stationary in the intervals D(x), where D(x) denotes the domain of local stationarity of f(x) containing the point x. (For simplicity, we confine ourselves to random functions of one variable, leaving to the reader the straightforward generalizations for random functions of several variables.) Then f(x) is locally ergodic in D(x) if the following two statements are true.

1. The ensemble average

$$< f(x) > = \int_S f(x,\alpha)\, d\mu(\alpha)$$

and the x-average

---

*See, for example, [10], p. 113.

$$\frac{1}{\ell\left[D(x)\right]}\int_{D(x)} f(\xi,\alpha)d\xi$$

are approximately equal for approximately all sample functions*. The second integral extends over the domain $D(x)$, of length $\ell\left[D(x)\right]$. Note that $< f(x) >$ is approximately independent of x in $D(x)$.

2. Let x and $x+x_o$ be two points in the same domain $D(x) = D(x+x_o)$ such that $\ell\left[D(x)\right] >> |x_o|$. Then the ensemble average

$$< f(x)f^*(x+x_o) > = \int_S f(x,\alpha)f^*(x+x_o,\alpha)d\mu(\alpha)$$

and the x-average

$$\frac{1}{\ell\left[D(x)\right]}\int_{D(x)} f(\xi,\alpha)f^*(\xi+x_o,\alpha)d\xi$$

are approximately equal for approximately all sample functions. Note that $< f(x)f^*(x+x_o) >$ is, to a good approximation, a function of x_o only in the domain $D(x)$.

Failure to define and use ensemble averages has led to some minor confusion in the radio scattering literature. The scattered radiation is a linear transform of the dielectric noise, involving an integration over $\vec{r}$ (see equation (16)). Thus, if the correlation function of the dielectric noise is defined as a space average, as by Booker and Gordon[11], a difficulty arises due to the double service required of the variable $\vec{r}$. This difficulty was noted by Staras[12], who prefers to define the correlation function as a time average. Since no integration over time appears in the usual approximation for the scattered signal, this may be considered a technical improvement. However, it is easy to imagine situations where time integrals are required, e.g., if the bandwidth of the incident radiation is so large that retardation effects cannot be neglected. One way to avoid these spurious difficulties is to define and use ensemble averages, and to invoke ergodicity only at the end of a calculation, when it is finally necessary to identify ensemble averages with appropriate space and time averages. On the other hand, it cannot be denied that there are versions of radio scattering theory that get along without the use of ensemble or phase averages.

---

* Just as almost all means all except for a set of zero μ-measure, approximately all means all except for a set of negligibly small μ-measure.

3. Some properties of dielectric turbulence

The relative dielectric constant in the atmosphere at the point $\vec{r}$ and time t is a second-order random function $\varepsilon(\vec{r},t)$, which is the result of many different kinds of dielectric noise, each with its own characteristic periodicities. The most prevalent kind seems to be dielectric turbulence at large Reynolds number, which in this paper will be assumed to be the only kind of dielectric noise present in the atmosphere. Thus, the sample functions $\varepsilon(\vec{r},t,\alpha)$, the sample space S, and the probability measure μ will be taken to correspond to outcomes of the process $\varepsilon(\vec{r},t)$ that differ because of the action of atmospheric turbulence, and departures of $\varepsilon(\vec{r},t)$ from stationarity will be ascribed to trends (both spatial and temporal) in the meterological mechanisms that produce the turbulence.

We now postulate certain properties of dielectric turbulence that are suggested by experiment. We admit in advance that the complexities of meteorology are such that the best that can be hoped for is that these postulates are an approximation to the true state of affairs most of the time and in large regions of the atmosphere.

1. For each fixed $\vec{r}$, $\varepsilon(\vec{r},t)$ is locally t-stationary in time domains $D_r(t)$ of the order of several minutes*. (Airborne refractometer measurements in the troposphere show that this assumption is usually justified; in the case of the ionosphere the validity of the assumption is more hypothetical, due to the lack of direct measurements.) On the other hand, $\varepsilon(\vec{r},t)$ is not locally r-stationary in domains large enough to make the property useful, as shown by the spatial changes in $< \varepsilon(\vec{r},t) >$ associated with the normally stratified atmosphere, not to mention the abrupt changes associated with sharp elevated layers.

2. The centered second-order random function $\triangle\varepsilon(\vec{r},t) = \varepsilon(\vec{r},t) - < \varepsilon(\vec{r},t) >$ is locally t-stationary in the same domains $D_r(t)$ as $\varepsilon(\vec{r},t)$, i.e., in time domains

---

*As we shall see below, in the radio scattering problem we need only assume second-order local stationarity, but we shall not always state this explicitly.

of the order of several minutes. This follows from the fact that the operation of centering is locally homogeneous with respect to t in the domains $D_r(t)$, since $< \varepsilon(\vec{r},t) >$ is approximately independent of t in $D_r(t)$. Note that by definition $< \triangle\, \varepsilon(\vec{r},t) > = 0$.

3. For each fixed t, $\triangle\, \varepsilon(\vec{r},t)$ is locally r-stationary in space domains $D_t(\vec{r})$ with linear dimensions at least as large as 100 meters, and often considerably larger. The validity of this assumption is indicated by refractometer measurements in the troposphere, but, again, the situation in the ionosphere is more conjectural. The fact that $\triangle\, \varepsilon(\vec{r},t)$ is locally r-stationary in sizeable domains, although $\varepsilon(\vec{r},t)$ is not, is easily explained if $\triangle\, \varepsilon(\vec{r},t)$ is produced by turbulent mixing operating in gradients of dielectric constant. In this case the rms dielectric noise power $< \triangle\, \varepsilon^2(\vec{r},t) >^{1/2}$ is proportional to the gradient of the mean dielectric constant field [2], [4] so that*

$$(5) \qquad < \triangle\, \varepsilon^2(\vec{r},t) >^{1/2} \sim L_o \text{ grad} < \varepsilon(\vec{r},t) > .$$

The constant of proportionality L_o is the largest eddy size of the turbulent velocity field**. According to equation (5), the presence of gradients of $< \varepsilon(\vec{r},t) >$, which precludes the local r-stationarity of $\varepsilon(\vec{r},t)$, is perfectly compatible with the local r-stationarity of $\triangle\, \varepsilon(\vec{r},t)$, provided that there are sizeable domains in which grad $< \varepsilon(\vec{r},t) >$ is substantially constant, and in which the velocity field itself is locally r-stationary, as seems to be the case. The space-time domains in which $\triangle\, \varepsilon(\vec{r},t)$ is locally stationary are approximately the product sets

$$D(\vec{r},t) = D_t(\vec{r}) \times D_r(t) \quad .$$

---

*We shall use the symbol ~ to mean " approximately equal to" , sometimes (as here) only to within an order of magnitude; ~ is admittedly a symbol with a somewhat variable meaning, but this disadvantage is offset by its convenience in the radio scattering problem, where a large number of approximations are made.

**For background in turbulence theory, see G.K. Batchelor [13].

In radio scattering theory, the received signal is attributed to the centered random function $\triangle\, \varepsilon(\vec{r},t)$, which represents turbulent fluctuations of the dielectric constant about its mean . The amplitude of the current in the receiving antenna load is a random function of time I(t), which is a weighted Fourier space-transform of $\triangle\, \varepsilon(\vec{r},t)$, taken over a finite region of space V (see equation (16)). Since the transformation I(t) is homogeneous with respect to time, it follows that I(t) is locally t-stationary in the domains D(t), whose dimensions $\ell\,[D(t)]$ are given by*

$$\ell\,[D(t)] \;=\; \mathrm{Min}\; \ell\left[D_r(t)\right], \qquad \vec{r} \text{ in } V,$$

where $D_r(t)$ are the domains of local t-stationarity of $\triangle\, \varepsilon(\vec{r},t)$, with dimensions $\ell\,[D_r(t)]$. If all the domains $D_r(t)$ have dimensions of the order of a few minutes, then so do the domains D(t). The local t-stationarity of I(t) in domains of the order of a few minutes is an experimental fact. (This does not imply, but lends plausibility to the local t-stationarity of $\triangle\, \varepsilon(\vec{r},t)$ in domains of the same size.) For example, in the experiments of Chisholm et al [14], the envelope distribution of the received signal was found to be approximately Rayleigh if four-minute samples were analyzed, but approximately gaussian if 16-minute samples were analyzed. This change in distribution can be interpreted as a slow change in the parameter of the Rayleigh distribution, which describes the envelope of the scattered signal for several minutes at a time.

---

*If x is a typical point of a set E, and f a function of x, then the minimum value of f at a point of E, and the maximum value of f at a point of E, will be denoted by Min f(x), x in E, and Max f(x), x in E, respectively.

4. The process $\Delta\varepsilon(\vec{r},t)$ is characterized by a correlation distance L_o, which is the linear dimension of the region within which values of $\Delta\varepsilon(\vec{r},t)$ are appreciably correlated. As the symbol L_o indicates, the correlation distance is a way of characterizing the macro-eddy dimension, which is inherently not defined to within much better than an order of magnitude. One might, for example, estimate L_o as the distance such that the correlation between the random variables $\Delta\varepsilon(\vec{r},t)$ and $\Delta\varepsilon(\vec{r}\,',t)$ is $\leq 1/10$, provided that $|\vec{r}-\vec{r}\,'| \geq L_o$. Refractometer measurements in the troposphere indicate that $L_o \sim 100$ meters*; estimates of L_o in the ionosphere vary, but the value $L_o \sim 100$ meters is favored by some authors, e.g., Gallet[4]. In analogy with the situation for the turbulent velocity field (see below) it is reasonable to assume that the joint distribution of the random variables $\Delta\varepsilon(\vec{r},t)$ and $\Delta\varepsilon(\vec{r}\,',t)$ is approximately gaussian provided that $|\vec{r}-\vec{r}\,'| \geq L_o$; thus, to a good approximation, $\Delta\varepsilon(\vec{r},t)$ and $\Delta\varepsilon(\vec{r}\,',t)$ are independent as well as uncorrelated if $|\vec{r}-\vec{r}\,'| \geq L_o$. By a correlation volume is meant a cube of side L_o.

5. As just noted, the maximum spatial periodicity (macro-eddy size) of $\Delta\varepsilon(\vec{r},t)$ has been estimated as $L_o \sim 100$ meters, in both the ionosphere and troposphere; the corresponding maximum temporal periodicity (macro-eddy lifetime) is $T_o \sim L_o/v_o \sim 2$ minutes, where v_o is the rms turbulent velocity (macro-eddy velocity), estimated at 1 meter/sec [4], [16]**. Molecular dissipation processes determine corresponding minimum periodicities. The minimum spatial periodicity (" smallest eddy") of dielectric turbulence is approximately the same as that of the underlying velocity field. It has been estimated as

---

*$L_o \sim \ell$, where ℓ is the parameter in the correction function $\exp(-|r|/\ell)$, used by Booker and Gordon[11]. Estimates of ℓ are summarized by Gordon[15].

**Since the smallest unit of dielectric turbulence is a single macro-eddy, L_o and T_o are estimated lower bounds for the space and time dimensions of the domains of local stationarity of $\Delta\varepsilon(\vec{r},t)$.

$$\eta \sim 1 \text{ cm (troposphere)}^{[16],[17]}$$

(6)

$$\eta \sim 10 \text{ meters (ionosphere)}^{*} .$$

From the standpoint of the radio scattering problem, the minimum temporal periodicity of dielectric turbulence corresponds to the case where the mean velocity field $< v >$ convects the spatial pattern of the smallest eddies past a point fixed with respect to the earth. This leads to the estimate

$$\tau_{min} \sim \eta / < v > .$$

Since the mean wind velocity certainly never exceeds 100 meters/sec in either the ionosphere or the troposphere, we find with the use of (6) that

$$\tau_{min} \gtrsim 10^{-4} \text{ sec (troposphere)},$$

$$\tau_{min} \gtrsim 10^{-1} \text{ sec (ionosphere)} .$$

If $< v > < v_o$, which can happen only if the turbulence arises from causes other than shear flow, then $< v >$ should be replaced by v_o in the expression for τ_{min}, corresponding to the convection of the smallest eddies by the largest eddies. This results in estimates of τ_{min} which are about 100 times larger than the above values. These considerations are enough to show that the fading of the scattered signal involves both the lifetimes of the scattering eddies and their convection by the mean wind or by the macro-eddies .

---

[*] This value of η is obtained by using the estimates $L_o \sim 100$ meters and $v_o \sim 1$ meter/sec; η is only weakly dependent on L_o, so that Villars and Weisskopf [3] arrive at the same estimate of η, using a much larger value of L_o. A value of η of this order of magnitude follows from an analysis of the scaled radio scattering experiments of Bailey, Bateman, and Kirby [18], as pointed out by Wheelon [19].

6. We must now postulate a plausible ergodic property in order to identify ensemble averages of $\triangle \varepsilon(\vec{r},t)$ and certain of its transforms with suitable space and time averages. A cautionary remark is needed before invoking a local ergodic property like that described in Section 2, since, as already noted, it is conceivable that the dimensions of some of the domains of local stationarity of the process $\triangle \varepsilon(\vec{r},t)$ may be of the same order of magnitude as the macro-eddy size L_o and the macro-eddy lifetime T_o. It is clear that sample functions measured in such a domain, with dimensions L_o and T_o, will not be large enough to exhibit typical behavior of the process insofar as events on the space-time scale of the macro-eddies are concerned. To avoid this difficulty we must take care to apply the local ergodic property only to transforms of $\triangle \varepsilon(\vec{r},t)$ which are substantially unaffected by macro-eddy behavior. This point of view is especially cogent, when it is recalled that description of events on the space-time scale of the macro-eddies lies outside the province of statistical turbulence theory, which by its very nature can make statements only about the local structure of turbulence.* Kolmogoroff made this fact explicit in his original researches on turbulence[9] by introducing the structure function

$$D_v(\vec{r}\,') = < |v(\vec{r},t) - v(\vec{r} + \vec{r}\,',t)|^2 >,$$

where v denotes some component of the turbulent velocity field. Statements about $D_v(\vec{r}\,')$ are then made only for $|\vec{r}\,'| << L_o$, so that the effects of the macro-eddies are automatically subtracted out. The amplitude $I(t)$ of the received scattered current is another important example of a transform of the process $\triangle \varepsilon(\vec{r},t)$ which is insensitive to macro-eddy events, since it is produced by scattering eddies at least an order of magnitude smaller than L_o. Correspondingly, the largest periodicities of $I(t)$ are much less than T_o. With these qualifications, we shall assume a local ergodic property of the type discussed in Section 2, whenever needed.

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*In fact, it is often difficult to distinguish between macro-eddy events and the meteorological events responsible for the turbulence.

7. It is sometimes assumed that $\triangle\,\varepsilon(\vec{r},t)$ is a gaussian process. There is very little likelihood that this is the case. Since the turbulent velocity field is not a gaussian process*, other turbulent fields should be non-gaussian to the extent that they share the structure of the underlying velocity field. Therefore, it is reasonable to expect that atmospheric dielectric noise, which is due primarily to the turbulent mixing of dynamically passive** scalar quantities will be found to be non-gaussian***. Presumably, departures from the gaussian distribution will first show up in the bivariate distribution of $\triangle\,\varepsilon(\vec{r},t)$ at points separated by less than L_0, in analogy with the situation for the turbulent velocity field. It is recommended that refractometer data be analyzed with a view to studying the probability distribution of $\triangle\,\varepsilon(\vec{r},t)$; this does not seem to have been done as yet.

The reader will note that our model represents turbulent fluctuations about the actual mean dielectric constant field $<\varepsilon(\vec{r},t)>$, rather than about the vacuum value of unity, as often written. We have chosen this model for two reasons.

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* Although the turbulent velocity field is univariate gaussian, its bivariate distribution is markedly non-gaussian for small spatial separations, but becomes gaussian when the points are far enough apart so that the corresponding two-point velocity correlation is quite small. See Batchelor[13], chapter 8. A gaussian velocity field could not represent turbulence, since the vanishing of its third moments would preclude the transfer of energy from larger to smaller eddies characteristic of turbulent flow. The non-gaussian character of the velocity field corresponds to statistical dependence between the Fourier coefficients of the velocity field.

**A turbulent field quantity like temperature is said to be dynamically passive (in a given turbulent regime) if any turbulence produced by its inhomogeneities is negligible compared to the turbulence produced by other causes.

***As shown below, departures of $\triangle\,\varepsilon(\vec{r},t)$ from the gaussian distribution do not affect the scattered signal. Thus, if we are not concerned with the detailed structure of the dielectric turbulence, we can replace $\triangle\,\varepsilon(\vec{r},t)$ by the zero-mean gaussian process with the same covariance, without in any way changing the scattered signal.

First of all, since we favor turbulent mixing operating in pre-existing inhomogeneities of dielectric constant as the source of most atmospheric dielectric noise, it is desirable to exhibit the term $< \varepsilon(\vec{r},t) >$, which explicitly represents the inhomogeneities of dielectric constant. Secondly, although we do not attribute the received signal to $< \varepsilon(\vec{r},t) >$, it is not to be denied that the mean dielectric constant field has an effect on radio scattering, an effect conventionally concealed in the antenna pattern propagation factors. Writing out the term $< \varepsilon(\vec{r},t) >$ serves as a reminder that one must justify neglecting the departure of $< \varepsilon(\vec{r},t) >$ from unity at any stage of a calculation, especially since the departure of $< \varepsilon(\vec{r},t) >$ from unity is usually much greater than the largest fluctuations of dielectric constant produced by turbulence.

4. Harmonic analysis of $\triangle\, \varepsilon(\vec{r},t)$

Of particular interest in physical problems is the representation of random functions as Fourier-Stieltjes integrals. Accordingly, we shall assume that $\triangle\, \varepsilon(\vec{r},t)$ is harmonizable, i.e., that there exists a second-order random function $g(\vec{k},\omega)$, with a covariance of bounded variation, such that*

$$(7) \qquad \triangle\, \varepsilon(\vec{r},t) = \int dg(\vec{k},\omega)\, \exp\left[-i(\vec{k}\cdot\vec{r} + \omega t)\right], \quad (\text{q.m.})$$

where all variables of integration range from $-\infty$ to ∞, as they will in any integral that we write without limits. Equation (7) is a stochastic integral, in the sense that it corresponds to the limit of a sequence of random variables. Let $S_n(\vec{r},t)$ be a suitable approximation to the stochastic integral which is the sum of n random variables. The abbreviation (q.m.) appearing in (7) signifies that the convergence of the sequence $S_n(\vec{r},t)$ to $\triangle\,\varepsilon(\vec{r},t)$ is meant in the sense of the quadratic mean, i.e., that

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* For a discussion of harmonizability, see Loève [7] p. 474. Note that we use a minus sign in the complex exponential instead of the more usual plus sign. Stochastic integrals of the type used here, i.e., Riemann-Stieltjes integrals in the quadratic mean, are discussed in [7], p. 472.

$$\lim_{n\to\infty} < |\triangle\,\varepsilon(\vec{r},t) - S_n(\vec{r},t)|^2> \; = \; 0,$$

for every $\vec{r}$ and t. Of the reasons for this stipulation, perhaps the most cogent is that the integral (7) may not exist for the individual realizations of $g(\vec{k},\omega)$*. It follows from the fact that $\triangle\,\varepsilon(\vec{r},t)$ is a real-valued random function that

$$dg^*(\vec{k},\omega) \; = \; dg(-\vec{k},-\omega);$$

moreover, $< dg(\vec{k},\omega) > = 0$, since $< \triangle\,\varepsilon(\vec{r},t) > \; = \; 0$.

Since we intend to approximate $\triangle\,\varepsilon(\vec{r},t)$ in its domains of local stationarity by pieces of suitable globally stationary random functions, as discussed in section 2, it is enough to consider the case where the covariance of $\triangle\,\varepsilon(\vec{r},t)$ is globally stationary in $\vec{r}$ and t. Then, the covariance of $\triangle\,\varepsilon(\vec{r},t)$ is a function only of the coordinate differences $\vec{\rho} = \vec{r}-\vec{r}\,'$ and $\tau = t-t'$, i.e.,

$$< \triangle\,\varepsilon(\vec{r},t)\, \triangle\,\varepsilon(\vec{r}\,',t') > \; = \; < \triangle\,\varepsilon^2 > R(\vec{\rho},\tau),$$

where $R(\vec{\rho},\tau)$ is the space-time correlation function of $\triangle\,\varepsilon(\vec{r},t)$. Furthermore, by a well-known theorem of probability theory (see [7], p. 483), the covariance of $dg(\vec{k},\omega)$ is of the form

$$(8) \qquad < dg(\vec{k},\omega)dg^*(\vec{k}\,',\omega') > \; = \; < \triangle\,\varepsilon^2> \Psi(\vec{k},\omega)\delta(\vec{k}-\vec{k}\,')\delta(\omega-\omega')d\vec{k}d\vec{k}\,'d\omega d\omega'.$$

where the non-negative function $\Psi(\vec{k},\omega)$ is the space-time spectral density of $\triangle\,\varepsilon(\vec{r},t)$, measured in units of the variance $< \triangle\,\varepsilon^2 >$, so that

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* If the process $\triangle\,\varepsilon(\vec{r},t)$ is (globally) stationary, almost all realizations of $g(\vec{k},\omega)$ are of unbounded variation, so that almost all the Riemann-Stieltjes integrals of the individual realizations of $g(\vec{k},\omega)$ fail to exist.

$$\int \Psi(\vec{k},\omega)\, d\vec{k}\, d\omega = 1.$$

(We can assume that the process $\Delta\varepsilon(\vec{r},t)$ has no line spectrum, an assumption which can be readily justified for the turbulent velocity field (see [13], pp. 25, 85), and, presumably, for dielectric turbulence as well.) It is easily seen that $R(\vec{\rho},\tau)$ and $\Psi(\vec{k},\omega)$ are Fourier-transform pairs, i.e., that

$$R(\vec{\rho},\tau) = \int \Psi(\vec{k},\omega) \exp\left[-i(\vec{k}\cdot\vec{\rho} + \omega\tau)\right] d\vec{k}\, d\omega ,$$

$$\Psi(\vec{k},\omega) = 1/(2\pi)^4 \int R(\vec{\rho},\tau) \exp\left[i(\vec{k}\cdot\vec{\rho} + \omega\tau)\right] d\vec{\rho}\, d\tau .$$

As already noted, finite Fourier space-transforms of the process $\Delta\varepsilon(\vec{r},t)$ are of great importance in radio scattering theory. Accordingly, we introduce the harmonic decomposition

$$(9) \qquad \Delta\varepsilon(\vec{r},t) = \int dg_1(\vec{k},t) \exp(-i\vec{k}\cdot\vec{r}), \qquad (q.m.)$$

corresponding to spatial harmonic analysis of $\Delta\varepsilon(\vec{r},t)$ at the time t. The covariance of $dg_1(\vec{k},t)$ is

$$(10) \qquad < dg_1(\vec{k},t)dg_1^*(\vec{k}',t') > = < \Delta\varepsilon^2 > \Phi(\vec{k},t-t')\delta(\vec{k}-\vec{k}')d\vec{k}\, d\vec{k}' ,$$

where, as is easily verified

$$\Phi(\vec{k},\tau) = \int \Psi(\vec{k},\omega)\exp(-i\omega\tau)d\tau = 1/(2\pi)^3 \int R(\vec{\rho},\tau)\exp(i\vec{k}\cdot\vec{\rho})d\vec{\rho} .$$

The mixed quantity $\Phi(\vec{k},\tau)$ is a spectral density in the variable $\vec{k}$, and an unnormalized correlation function in the variable $\tau = t-t'$. Consider now the finite Fourier space-transform

$$J(d,\vec{K};t) = \int_V \triangle\,\varepsilon(\vec{r},t)\,\exp(i\vec{K}\cdot\vec{r})\,d\vec{r}\,, \tag{11}$$

where V is a cube of side d, centered at the origin; $J(d,\vec{K};t)$ is a transform of $\triangle\,\varepsilon(\vec{r},t)$ of the type of equation (4). Substituting (9) in(11), we obtain

$$J(d,\vec{K};t) = V \int \operatorname{sinc}\left[(\vec{K}-\vec{k})d/2\right] dg_1(\vec{k},t), \quad (\text{q.m.}) \tag{12}$$

where sinc $\vec{k}$ is the function

$$\operatorname{sinc}\vec{k} = (\sin k_x/k_x)(\sin k_y/k_y)(\sin k_z/k_z).$$

Using (10), we find the following expression for the covariance of $J(d,\vec{K};t)$

$$< J(d,\vec{K};t)J^*(d,\vec{K};t') > = V^2 < \triangle\,\varepsilon^2 > \int \operatorname{sinc}^2\left[(\vec{K}-\vec{k})d/2\right] \Phi(\vec{k},\tau)d\vec{k}\,, \tag{13}$$

where $\tau = t-t'$.

The function sinc $\vec{k}$ has the property that

$$\int \operatorname{sinc}\vec{k}\,d\vec{k} = \int \operatorname{sinc}^2\vec{k}\,d\vec{k} = \pi^3 \tag{14}$$

Moreover, most of the contribution to the integrals (14) comes from a region of volume π^3 in k-space, centered about the origin. Correspondingly, most of the contribution to (13) comes from the volume $V' = (2\pi)^3/V$ in k-space, centered about the point $\vec{K}$, so that we can write

$$< J(d,\vec{K};t)J^*(d,\vec{K};t') > \sim (2\pi)^3\,V < \triangle\,\varepsilon^2 > \Phi(\vec{K},\tau) \tag{15}$$

to a good approximation, provided that the following two qualitative conditions are met. (Note that as $d \rightarrow \infty$, the approximation (15) becomes an exact equality.)

1. For every τ, $\Phi(\vec{k},\tau)$ must be substantially constant in the volume V' centered about $\vec{K}$. For any reasonable shape of $\Phi(\vec{k},\tau)$, this means that $K + 2\pi/d \sim K$, where $K = |\vec{K}|$. If we divide both sides of this approximate equality by K, it

becomes $2\pi/Kd << 1$, or $Kd >> 2\pi$. This condition, i.e., that the linear dimension of the finite sample of $\Delta\,\varepsilon(\vec{r},t)$ be long compared to the wavelength at which the spectrum is being estimated, was noted by Batchelor [16].

2. $\Phi(\vec{k},\tau)$ must not become so large away from the point $\vec{K}$ that most of the contribution to the covariance of $J(d,\vec{K};t)$ comes from a region of k-space lying outside the volume V' centered about $\vec{K}$. If this condition is insured by the first condition, namely $Kd >> 2\pi$, then we shall say that $\Delta\,\varepsilon(\vec{r},t)$ has the local harmonic property (in $\vec{r}$), or that its spectral density has the local harmonic property (in $\vec{k}$). It should be noted that the approximation (15) is always good if $d >> L_o$; it is assumed, of course, that there are values of $d << L_o$, which satisfy the relation $Kd >> 2\pi$. Since $\Phi(\vec{k},\tau)$ is a correlation function in the variable τ, it follows that $\Phi(\vec{k},\tau) \leq \Phi(\vec{k},0)$, so that if the space spectral density $\Phi(\vec{k},0)$ has the local harmonic property, so should the "mixed" spectral density $\Phi(\vec{k},\tau)$. There are, of course, random functions without the local harmonic property*. However, it is intuitively evident that turbulent fields in general, and dielectric turbulence in particular, have the local harmonic property. For in the picturesque but meaningful language of statistical turbulence theory, harmonic content at wave number K corresponds to the presence of eddies of size $\sim 2\pi/K$, which are only weakly coupled to eddies of appreciably different sizes**. It follows that a "piece" of turbulence large enough to contain many eddies of size $\sim 2\pi/K$ should be large enough to determine the spectral density of the turbulence about wave number K, i.e., that the turbulence has the local harmonic property. Thus, we shall assume that (15) is a good approximation if $\frac{Kd}{2\pi} \gtrsim 10$, say; Batchelor [16] makes the same assumption, although less explicitly.

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* For example, the output of an RC filter with a white noise input does not have the local harmonic property.

** A basic assumption of statistical turbulence theory is that eddies of a given size "forget" the eddies of much larger size which give rise to them; if this were not the case, one would hardly be justified in constructing a statistical or probabilistic model of turbulence.

It is not to be denied that these heuristics should be backed up by some rigorous mathematics. The author has begun this study, and plans to present the results in a future paper.

5. Relation between the scattered signal and $\Delta\,\varepsilon(\vec{r},t)$

The geometrical configuration of a radio scattering experiment consists of a transmitting antenna A emitting electromagnetic radiation that is scattered into a receiving antenna A' by dielectric noise in the common volume of the antenna beams. Before writing the approximation for the received scattered current, we give our versions of some rather standard nomenclature and symbols.

1. c is the velocity of light; ω_0, the angular carrier frequency; $k = \omega_0/c$, the carrier wave number; $\lambda = 2\pi c/\omega_0$, the carrier wavelength.

2. D is the distance from A to A' along the earth's surface*; $R(\vec{r})$, the distance from A to the variable point $\vec{r}$; $R'(\vec{r})$, the distance from A' to $\vec{r}$.

3. $\theta(\vec{r}),\phi(\vec{r})$ are the polar angles of $\vec{r}$ referred to A as origin; $\theta'(\vec{r})$, $\phi'(\vec{r})$, the polar angles of $\vec{r}$ referred to A' as origin; $\vec{n}(\vec{r})$, the unit vector with polar angles $\theta(\vec{r}),\phi(\vec{r})$ pointing away from A; $\vec{n}'(\vec{r})$, the unit vector with polar angles $\theta'(\vec{r})$, $\phi'(\vec{r})$ pointing toward A'.

4. $\vec{K}(\vec{r})$ is the scattering wave vector associated with the point $\vec{r}$, i.e., the quantity

$$\vec{K}(\vec{r}) = (2\pi/\lambda)\left[\vec{n}'(\vec{r}) - \vec{n}(\vec{r})\right] ;$$

$\Theta(\vec{r})$ is the scattering angle associated with the point $\vec{r}$, i.e., the angle between $\vec{n}(\vec{r})$ and $\vec{n}'(\vec{r})$. For the usual small scattering angles, the magnitude of $\vec{K}(\vec{r})$ can be written as

$$K(\vec{r}) \sim 2\pi\,\Theta(\vec{r})/\lambda \ .$$

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* We assume that A' is well beyond the geometrical horizon from A, i.e., that there is no optical path between A and A'. The effect of dielectric noise on line-of-sight propagation is a problem requiring its own special methods, and will not concern us here.

5. $f(\theta,\emptyset)$ and $f'(\theta',\emptyset')$ are the pattern propagation factors of A and A' *; $F(\vec{r})$, the joint pattern propagation factor associated with the point $\vec{r}$, defined by

$$F(\vec{r}) = f\left[\theta(\vec{r}), \emptyset(\vec{r})\right] \quad f'\left[\theta'(\vec{r}), \emptyset'(\vec{r})\right] .$$

We assume that A and A' are sited so that their directions of maximum gain intersect in a point $\vec{r}_c$, at which $F(\vec{r}_c) = 1$. We also assume that both A and A' produce linearly polarized radiation, and are sited with their polarization directions parallel.

6. The region of space where $F(\vec{r})$ is substantially different from zero is called the (nominal) scattering volume V, and the point $\vec{r}_c$ is called the center of V. Although V serves to indicate the region of space responsible for the received scattered radiation, most of the radiation may come from a subset of V, known as the effective scattering volume (see below).

7. Θ_{min} is the minimum scattering angle. Using the " four-thirds earth" approximation to estimate the effect of atmospheric refraction (see [20], p. 53), we have $\Theta_{min} \sim 3D/4a$, where a is the radius of the earth.

We now write an approximation for the signal scattered by weak dielectric noise which has become standard since its introduction by Booker and Gordon[11]. It states that the complex amplitude of the current in the receiving antenna load, measured in units of the amplitude of the load current that would be found if the antennas were in free space and oriented with their directions of maximum gain along the line joining them, is given, to a good approximation, by

$$(16) \qquad I(t) \sim (\pi D/\lambda^2) \int_V \frac{F(\vec{r})}{R(\vec{r})R'(\vec{r})} \triangle \varepsilon(\vec{r},t) \exp\left[i\vec{K}(\vec{r})\cdot\vec{r}\right] d\vec{r} \ ;$$

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* By the pattern propagation factor is meant the quantity defined by Kerr ([20], p. 28). It is the analog of the antenna pattern function ([20],p.34), and, in principle, incorporates all the propagation effects of the actual surroundings of the antenna, including reflections from the earth's surface, and refraction in the earth's atmosphere. In practice, rough approximations are used, due to the difficulty of ascertaining precisely the 'surroundings' of the antenna.

the integral is over the scattering volume V, and an irrelevant phase factor has been omitted. The interpretation of (16) is that I(t) is a complex random function of time, related to $\triangle\,\varepsilon(\vec{r},t)$ by a transformation of the type (4). Explicitly we have

$$I(t) = I'(t) + i\,I''(t),$$

where $I'(t)$ and $I''(t)$ are the real and imaginary parts of $I(t)$. From the fact that $< \triangle\,\varepsilon(\vec{r},t) > = 0$, it follows that $< I(t) > = 0$. On the other hand, the average envelope is

$$< |I(t)| > = < \sqrt{I'^2(t) + I''^2(t)} > \neq 0.$$

In the opinion of the author, none of the many derivations of equation (16) found in the radio scattering literature is sufficiently free of heuristics to be considered definitive. However, for the purposes of this paper, we shall regard the familiar formula (16) as a correct result. Two of the many assumptions on which the validity of the approximation (16) rests are worth noting. One is that the maximum scattering angle is small, which permits the neglect of polarization effects. The other is that the spatial pattern of $\triangle\,\varepsilon(\vec{r},t)$ remains substantially unchanged in the longest time it takes the incident radiation to traverse the scattering volume V, which permits the neglect of retardation effects. We now describe two further approximations that can be applied to (16).

For an arbitrary (sure) function $h(\vec{r})$, consider the quantity

$$\delta h = \mathrm{Max}\left[h(\vec{r}) - h(\vec{r}\,')\right], \quad \vec{r}, \vec{r}\,' \text{ in } V.$$

Then if $\delta h << h(\vec{r}_c)$, where $\vec{r}_c$ is the center of V, it is a good approximation to replace $h(\vec{r})$ by the constant value $h(\vec{r}_c)$ in any integral over V. In particular, if the inequalities $\delta R << R(\vec{r}_c)$, $\delta R' << R'(\vec{r}_c)$, and $\delta K << K(\vec{r}_c)$ are valid, the functions $R(\vec{r})$, $R'(\vec{r})$ and $\vec{K}(\vec{r})$ can be replaced by their values at $\vec{r}_c$ in equation (16), an approximation which we shall call the Fraunhofer approximation, for an evident reason. The Fraunhofer approximation is a good approximation in some radio scattering experiments and not in others, as the following simple considerations show.

Consider a scattering experiment with path length D and with identical antennas of half-power beamwidth β. Then $R(\vec{r}_c) \sim R'(\vec{r}_c) \sim D/2$, and $\delta R \sim \delta R' \sim (\pi\beta/2\,\Theta_{min})$, so that the conditions $\delta R << R(\vec{r}_c)$ and $\delta R' << R'(\vec{r}_c)$ become $\beta << \Theta_{min}$. Moreover, $\Theta(\vec{r}_c) \sim \Theta_{min} + \beta$, and $\delta\Theta \sim 2\beta$, so that the conditions $\delta\Theta << \Theta(\vec{r}_c)$ also reduces to the inequality $\beta << \Theta_{min}$. As we have noted, $\Theta_{min} \sim D/a$, so that this inequality is reasonably valid for long distance experiments, using high gain antennas. However, for short distance tropospheric experiments, the inequality may fail completely, e.g., if $\beta \sim 1/50$ radian and $D \sim 100$ km.

Another common approximation, which we shall call the uniform illumination approximation, for an evident reason, consists in setting $F(\vec{r}) = 1$, throughout V. The Fraunhofer and uniform illumination approximations can never be equally good. If the Fraunhofer approximation is good, there is considerable variation of $F(\vec{r})$ throughout V, by the very definition of V. On the other hand, if the Fraunhofer approximation fails, the variation of $\vec{K}(\vec{r})$ within V determines a subvolume of V which contributes most of the scattered radiation, and then the approximation $F(\vec{r}) \sim 1$ is good in this effective subvolume[21].

6. Statistical structure of the scattered signal

We turn now to a study of the statistical structure of $I(t)$. Of basic importance is the fact that $I(t)$ is a gaussian random function of time; this is shown as follows.

Consider a radio scattering experiment with path length D, identical antennas of half-power beamwidth β, and typical scattering angle Θ. Then the scattering volume V can be estimated as

$$V \sim D^3\,\beta^3/8\,\Theta\,.$$

Let N be the ratio of V to a correlation volume, i.e.,

$$N = V/L_o^3 \sim (1/8\,\Theta)(D\beta/L_o)^3 .$$

In any ratio scattering experiment $N >> 1$; for example, if $\beta \sim 1/100$ radian and $D \sim 200$ km, then $N \sim 10^4$. We now divide V into subvolumes $V_1,\ldots,V_N$; for simplicity and without loss of generality, we imagine all these subvolumes to be cubes of side L_o. Let the contribution to $I(t)$ from the volume V_i be denoted by $I_i(t)$, so that

$$I(t) = \sum_{i=1}^{N} I_i(t) .$$

$I_i(t)$ is, of course, given by equation (16) with the volume of integration replaced by V_i. For every fixed t, the random variable $I_i(t)$ is substantially independent of the random variable $I_j(t)$, $j \neq i$, because of our choice of the volumes V_i, unless V_i and V_j are adjacent, in which case there is a weak dependence between $I_i(t)$ and $I_j(t)$. It follows by the central limit argument that, for every t, $I(t)$ is approximately gaussian. Indeed, by the n-dimensional central limit argument, the joint distribution of the random variables $I(t_1),\ldots,I(t_n)$, where $n << N$ and $t_1,\ldots,t_n$ are any n instants of time, is approximately gaussian, i.e., $I(t)$ is approximately a gaussian random function of time*. The received scattered signal looks like narrow-band gaussian noise, and numerous experiments confirm the fact that the univariate distribution of $I(t)$ is gaussian, i.e., that the univariate distribution of $|I(t)|$ is Rayleigh[14], [23]; the author knows of no experiment that tests the multivariate gaussian nature of $I(t)$.

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*By the central limit argument we mean the qualitative statement that the sum of a large number N of random variables is approximately gaussian if the dependence between the summands is sufficiently weak and if no small group of summands contributes most of the variance of the sum. (In the application given here, we assume that the weighting factors $F(\vec{r})$, $R(\vec{r})$, $R'(\vec{r})$, etc., do not vary enough to inhibit the operation of the central limit argument.) This rough statement is supported by a highly developed central limit theory, the sophistication of which far transcends the ordinary needs of physicists and engineers. By the n-dimensional central limit argument, we mean the extension of the ordinary central limit argument to the case of n-dimensional vectors, where $n << N$.

The eddy heuristics that lead us to believe that dielectric turbulence has the local harmonic property suggest that the subvolumes used in the central limit argument need only have linear dimension d, where $K^{-1} << d << L_o$. (In most scattering experiments, such values of d can be found.) If so, this would result in more summands and a more powerful central limit argument. However, because of the usual large size of N, the cruder central limit argument, which uses correlation volumes, is quite satisfactory.

In some earlier work[5], [22], the author presented an argument purporting to show that I(t) is approximately gaussian if the dielectric noise originates in turbulent pressure fluctuations; the central limit argument was used to show that Fourier coefficients of the turbulent pressure field are gaussian random variables. However, the validity of this argument seems to be open to question*. In any event, this matter, although of some interest in turbulence theory, is of only academic interest in radio scattering theory, since there is now little doubt that the turbulent pressure field makes a negligible contribution to atmospheric dielectric noise.

Since I(t) is a zero-mean gaussian random function, then, as is well known, its statistical structure is completely specified by giving its covariance $< I(t)I^{*}(t') >$. Moreover, since I(t) is a linear transform of $\triangle \varepsilon(\vec{r},t)$, we need only know the covariance (or spectrum) of $\triangle \varepsilon(\vec{r},t)$ in order to calculate the covariance (or spectrum) of $I(t)$. Two consequences of this fact have already been mentioned, namely that all stationarity and ergodicity properties of $\triangle \varepsilon(\vec{r},t)$ need only be second-order, and, secondly, that $\triangle \varepsilon(\vec{r},t)$ can be replaced by any process with the same covariance without affecting the scattered signal; in particular, we can substitute the zero-mean gaussian process with the same covariance, which always exists (see [6], p. 74).

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* Fourier coefficients of the pressure field can be written as convolutory sums of the Fourier coefficients of the turbulent velocity field in a familiar manner[23], [24]. It is correctly observed in [5] that the velocity coefficients are not statistically independent as assumed in [22]. However, it is very difficult to ascertain whether the dependence between the summands is weak enough to meet the requirements of the dependent central limit argument, as assumed in [5]. The fact that the univariate distribution of the velocity field is gaussian, whereas its bivariate distribution at small separations is non-gaussian, shows how delicately poised such arguments are.

To complete our study of the statistical structure of the scattered signal, we now relate the covariance $< I(t)I^{*}(t') >$ to the mixed spectral density of the process $\triangle \varepsilon(\vec{r},t)$, defined in section 4. We proceed in two steps. First we make the Fraunhofer and uniform illumination approximations by setting $F(\vec{r}) = 1$, $R(\vec{r}) = R$, $R'(\vec{r}) = R'$, and $\vec{K}(\vec{r}) = \vec{K}$, where by R, R', and $\vec{K}$, we mean the values of the corresponding functions at $\vec{r}_c$, the center of the scattering volume. (We assume that the scattering volume is a cube of side d, centered at $\vec{r}_c$; clearly this entails no loss of generality.) We also assume that $\triangle \varepsilon(\vec{r},t)$ is locally stationary in the scattering volume V, so that it can be replaced in V by a " piece " of a globally stationary random function with the same covariance. (From now on it is understood that all stationarity requirements need only apply in the second-order sense.) With these assumptions it follows from (15) and (16) that

$$
\begin{aligned}
(17) \qquad < I(t)I^{*}(t') > &\sim (\pi D/\lambda^{2}RR')^{2} < J(d,\vec{K};t)J^{*}(d,\vec{K};t') > \\
&\sim (\pi D/\lambda^{2}RR')^{2} \; (2\pi)^{3} \, V < \triangle \varepsilon^{2} > \Phi(\vec{K},\tau),
\end{aligned}
$$

where $\tau = t-t'$.

Next we consider the case where the functions $F(\vec{r})$, $R(\vec{r})$, $R'(\vec{r})$, $\vec{K}(\vec{r})$ and the dielectric noise vary appreciably in the scattering volume V. For simplicity, we confine ourselves to a study of the average power $<|I|^{2}>$. As before we divide V into N subcubes of side L_o. It is easily seen that in volumes of side $L_o \sim 100$ meters, there is negligible percentage variation of the functions $F(\vec{r})$, $R(\vec{r})$, $R'(\vec{r})$, and $\vec{K}(\vec{r})$, regardless of whether the Fraunhofer or uniform illumination approximation applies in the total volume V. Thus, in each of the subvolumes V_i, the functions $F(\vec{r})$, $R(\vec{r})$, $R'(\vec{r})$, and $\vec{K}(\vec{r})$ can be replaced by their values at the center of V_i, which we denote by F_i, R_i, R_i', and $\vec{K}_i$. It follows that

$$(18) \qquad <|I|^2> \sim (\pi D/\lambda^2)^2 \sum_{i=1}^{N} (F_i/R_iR_i')^2 < J_i(L_o,\vec{K}_i;t)J_i^*(L_o,\vec{K}_i;t)> ,$$

where the subscript on J_i reminds us that it pertains to the dielectric noise in the subvolume V_i. (We neglect interactions between different J_i.)

To reduce (18) further, we recall that L_o is the least linear dimension of the domains of local stationarity of $\triangle\varepsilon(\vec{r},t)$ as well as its correlation distance, so that the volumes V_i are certainly domains of local stationarity of $\triangle\varepsilon(\vec{r},t)$. (Our subdivision of V is deliberately conservative; in most scattering experiments, the domains of local stationarity of $\triangle\varepsilon(\vec{r},t)$ can be expected to be much larger.) Thus $\triangle\varepsilon(\vec{r},t)$ can be approximated in each of the volumes V_i by a piece of a suitable globally stationary process. We now assume that for our purposes the change in dielectric noise from subvolume to subvolume can be regarded as due only to a change of the noise power $< \triangle\varepsilon^2(\vec{r}) >$, and not to a change in the normalized spectral density $\Phi(\vec{k},0)$.* Then, noting that in most radio scattering experiments $KL_o/2\pi \sim \Theta L_o/\lambda >> 1$, where Θ is a typical scattering angle, and assuming that dielectric turbulence has the local harmonic property (see Sect. 4), we can apply the approximation (15) to the covariance of J_i, so that (18) becomes

$$(19) \qquad <|I|^2> \sim (\pi D/\lambda^2)^2(2\pi)^3 \sum_{i=1}^{N} V_i(F_i/R_iR_i')^2 < \triangle\varepsilon^2 >_i \Phi(\vec{K}_i,0),$$

where $< \triangle\varepsilon^2 >_i$ denotes the value of $< \triangle\varepsilon^2(\vec{r}) >$ at the center of V_i. Equation (19) can be approximated by the integral

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* The plausibility of this assumption rests in part on the fact that we shall only require the spectrum of $\triangle\varepsilon(\vec{r},t)$ to be defined at wave numbers $\vec{K}$ satisfying $KL_o >> 2\pi$.

$$(20) \qquad <|I|^2> \sim (\pi D/\lambda^2)^2(2\pi)^3 \int_V \frac{F(\vec{r})}{R(\vec{r})R'(\vec{r})} < \Delta\, \varepsilon^2(\vec{r}) > \Phi\left[\vec{K}(\vec{r}),0\right] d\vec{r}.$$

If we define an *effective scattering volume* V_{eff} by

$$V_{eff} = \int_V F(\vec{r}) \frac{RR'}{R(\vec{r})R'(\vec{r})} \frac{< \Delta\, \varepsilon^2(\vec{r}) >}{< \Delta\, \varepsilon^2 >} \frac{\Phi\left[\vec{K}(\vec{r}),0\right]}{\Phi(\vec{K},0)} d\vec{r},$$

where $< \Delta\, \varepsilon^2 >$ is now the value of $< \Delta\, \varepsilon^2(\vec{r}) >$ at $\vec{r}_c$, then, when $\tau = 0$, (20) is just (17) with V replaced by V_{eff}. The usual rough estimates of the scattered power correspond to the approximation $V_{eff} \sim V$.

7. Conclusions

From the mathematical standpoint, much of radio scattering theory consists in a study of finite, weighted Fourier space-transforms of a certain random function of space and time, the *dielectric noise*. This transform is, of course, itself a random function of time, the *scattered signal*. Our purpose has been to give a unified treatment of the relation between these two random functions, expressed in the appropriate mathematical language. At points we have had to substitute plausible heuristics for rigor, because no theory of the *local* properties of second-order random functions, in particular local stationarity, and what we call the local harmonic property, has yet been developed. The complicated radio scattering problem is obviously not the context in which to attempt these extensions of random function theory.

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